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Non-Abelian Higher-Derivative Chern–Simons Theories

A. Foussats,[1] E. Manavella,[2] C. Repetto,[1] O. P. Zandron,[1] and O. S. Zandron[1]

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The canonical and path-integral quantization of the non-Abelian higher-derivative Chern–Simons model in three dimensions coupled to a matter field is constructed. The expression of the gauge field propagator in the momentum space for this higher-derivative model is computed. In the framework of the perturbative formalism, the diagrammatic and the Feynman rules are analyzed. Among other results, we conclude that higher-derivative terms added to the Lagrangian improve the ultraviolet behavior, rendering the model less divergent.

1. INTRODUCTION

The Chern–Simons term for both Abelian and non-Abelian cases has long been considered and is of increasing interest. It makes a strong impact on the physics of (2+1) dimensions, giving rise to several types of classical and quantum field theories (Jackiw, 1987; Siegel, 1979; Shonfeld, 1981; Deser *et al.*, 1982a,b, 1988; Hagen, 1984, 1985; Dzyaloshinskii *et al.*, 1988; Wiegmann, 1988; Polyakov, 1988; Bednorz and Müller, 1986; Anderson, 1987; Matsuyama, 1989, 1990a; Lüscher, 1989; Jackiw *et al.*, 1994; Lin and Ni, 1990; Avdeev *et al.*, 1992; Odintsov, 1992). In particular, two relevant cases are:

(i) The coupling of the CP^1 model with Chern–Simons theories (Dzyaloshinskii *et al.*, 1988; Wiegmann, 1988; Polyakov, 1988; Bowick *et al.*, 1986; Semenoff, 1988; Panigrahi *et al.*, 1988a,b; Babinovici *et al.*, 1984; Matsuyama, 1990b). The resulting model with an action of the hidden $U(1)$ gauge field was also used as a promising model to explain high-T_c superconductivity

[1] Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina.
[2] Facultad de Ciencias Exactas Ingeniería y Agrimensura de la UNR, 2000 Rosario, Argentina.

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phenomena (Bednorz and Müller, 1986; Anderson, 1987). Matsuyama (1990b) performed the canonical and path-integral quantizations of the fermion-coupled CP^1 model with the Abelian Chern–Simons term.

(ii) The use of the Chern–Simons term in the anyon formalism of quantum mechanics as well as in the nonrelativistic field theory (Wilczek, 1982a,b; Goldhaber, 1982; Jackiw and Redlich, 1983; Wu, 1984a,b; Arovas, 1985; Halperin, 1984; Cortés *et al.*, 1992, 1994).

Another crucial paper related to these topics is that of Witten (1988), in which the theoretical framework needed to understand knot theory was given.

On the other hand, several authors (Slavnov, 1971, 1977, 1981; Lee and Zinn-Justin, 1972; Ellis, 1975; Leon and Rodriguez, 1985; Kerstyen, 1988; Nesterenko, 1989; Li, 1991; Alvarez-Gaume *et al.*, 1990; Leibbrandt and Martin, 1992, 1994; Greco *et al.*, 1994; Foussats *et al.*, 1995) have investigated dynamical systems described by means of a singular Lagrangian density containing higher-derivative terms. For instance, the second-order formalism of the conformal (or superconformal) gravity (or supergravity) field theories is written in terms of a singular Lagrangian density which is a Chern–Simons term (or its supersymmetrization). Due to the constraints on curvatures, these are examples of proper higher-derivative theories (van Nieuwenhuizen, 1985; Foussats *et al.*, 1992). From the theoretical point of view, these kinds of theories present several interesting problems and constitute a current research area in quantum field theory. The lack of knowledge and difficulty in treating higher-derivative theories may be why they have not been intensively studied. Related to the higher-derivative character of the theory, one of the problems is connected with their unitarity (Hawking, 1987). The unitarity can be violated when ghost states with negative norm are present.

Another question is to analyze the regularization and renormalization problem. It is known that in a perturbative framework, the presence of higher-derivative terms improves the behavior of propagators at large momentum, rendering the theory less divergent (Alvarez-Gaume, 1990), which can be an interesting quality of the model. The case of Chern–Simons theory is believed to belong to the class of finite theory. That is to say, once the theory is regularized, a finite quantity is obtained without using the renormalization procedure. Of course, the price is the appearance of new vertices in the theory.

We believe in the importance of studying quantum methods in higher-derivative field theories. Therefore, the motivation of the present paper is to carry out the canonical and path integral quantization of a non-Abelian Chern–Simons theory containing higher-derivative terms in the action and coupled to a matter field. Later, it will be interesting to construct the perturbative theory for the model by defining proper Feynman rules and a suitable diagrammatic.

The paper is organized as follows. In Section 2, we construct the classical generalized Hamiltonian formalism, finding the first-class constraints associated with the gauge symmetries of the system. Next, the canonical quantization is carried out. In Section 3, the path integral method is developed by extending the Faddeev–Senjanovic formalism. Moreover, in opposition to what happens in the Abelian case, in the non-Abelian one, the determinant constructed with the gauge-fixing conditions and the first-class constraints depends on the gauge field variable. So, by means of the Faddeev–Popov trick it is shown how the ghost anticommuting auxiliary scalar fields must be introduced in this higher-derivative model. Finally, in Section 4, the diagrammatic and Feynman rules of the model are found. They are obtained by defining a suitable gauge field propagator.

2. GENERALIZED HAMILTONIAN FORMALISM AND CONSTRAINTS

With the aim of constructing the classical generalized Hamiltonian formalism and next of carrying out the corresponding canonical quantization, we will work as closely as possible to the Dirac (1964) algorithm.

We start by considering the following singular Lagrangian density:

$$\mathcal{L} = \mathcal{L}_{\text{top}} + \mathcal{L}_{\text{h}} + \mathcal{L}_{\text{f}} \tag{2.1}$$

describing the matter field ψ coupled to non-Abelian Chern–Simons (CS) theories in (2+1) dimensions whose $SU(N)$ gauge connection is called $\mathcal{A}_\mu$. So, the fields are written $\psi = \psi^a t^a$, $\mathcal{A}_\mu = A^a_\mu t^a$, and $\mathcal{F}_{\mu\nu} = F^a_{\mu\nu} t^a = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu + [\mathcal{A}_\mu, \mathcal{A}_\nu]$ (where an arbitrary coupling constant g in front of the commutator was omitted). The t^a are the generators of the Lie algebra associated to the gauge group $SU(N)$, i.e., $[t^a, t^b] = f^{abc} t^c$, $\text{tr}(t^a t^b) = \delta^{ab}$, $\text{tr}(t^a t^b t^c) = f^{abc}$, and a, b, c denote group representation indices. The field strength components are written

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu \tag{2.2}$$

To write the Lagrangian density, the trace in the Yang–Mills space must be performed.

In equation (2.1), $\mathcal{L}_{\text{top}}$ is the Lagrangian density for topologically massive $SU(N)$ gauge theory, i.e., a non-Abelian CS term, and it is given by

$$\begin{aligned}\mathcal{L}_{\text{top}} &= -\frac{1}{4}\,\text{tr}(\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}) + \frac{\kappa}{4\pi}\,\epsilon^{\mu\nu\rho}\text{tr}\left(\partial_\mu \mathcal{A}_\nu \mathcal{A}_\rho + \frac{2}{3}\,\mathcal{A}_\mu \mathcal{A}_\nu \mathcal{A}_\rho\right)\\ &= -\frac{1}{4}\,F^a_{\mu\nu}F^{a\mu\nu} + \frac{\kappa}{4\pi}\,\epsilon^{\mu\nu\rho}\left(\partial_\mu A^a_\nu A^a_\rho + \frac{1}{3} f^{abc} A^a_\mu A^b_\nu A^c_\rho\right)\end{aligned} \tag{2.3a}$$

The part containing higher derivatives is

$$\mathcal{L}_{\mathrm{h}} = -\frac{c^2}{4\pi}\mathcal{D}_{\rho}F^a_{\mu\nu}\mathcal{D}^{\rho}F^{a\mu\nu} \tag{2.3b}$$

and the fermionic piece is given by

$$\mathcal{L}_f = i\left(\frac{a+1}{2}\right)\bar{\psi}\gamma^{\mu}\mathcal{D}_{\mu}\psi + i\left(\frac{a-1}{2}\right)\mathcal{D}_{\mu}\bar{\psi}\gamma^{\mu}\psi - m\bar{\psi}\psi \tag{2.3c}$$

where the kinetic term of the fermion is included in the general form using the parameter a (Sundermeyer, 1982).

The covariant derivative $\mathcal{D}_{\mu}$ acting on objects V^a with a Yang–Mills index is $\mathcal{D}_{\mu}V^a = \partial_{\mu}V^a + f^{abc}A^b_{\mu}V^c$ and $[\mathcal{D}_{\mu}, \mathcal{D}_{\nu}]V^a = f^{abc}F^b_{\mu\nu}V^c$.

The constant κ is the topological mass of the gauge field and its dimension is $[\text{length}]^{-1}$; the dimensional constant c has dimension $[\text{length}]^1$. We will use the convention $\epsilon^{012} = \epsilon^{12} = 1$; the Minkowskian metric $g_{\mu\nu}$ is $g_{\mu\nu} = \text{diag}(1, -1, -1)$ and the Dirac γ-matrices are $\gamma^0 = \sigma^3$, $\gamma^1 = i\sigma^1$, and $\gamma^2 = i\sigma^2$ (σ's are the Pauli matrices).

Now we must define the canonical variables. When we have in hand the second-order Lagrangian density (2.1) the canonical variables are introduced according to the Ostrogradski (1850) transformation. Let us consider the independent dynamical field variables $\mathcal{A}_{\mu}$, $\mathcal{B}_{\mu} = \dot{\mathcal{A}}_{\mu}$, $\psi_{(\alpha)}$, and $\bar{\psi}_{(\alpha)}$ in the Lagrangian density (2.1). By performing the Ostrogradski transformation, the following canonical momenta are introduced:

$$\mathcal{P}^{\mu} = \frac{\partial\mathcal{L}}{\partial\dot{\mathcal{A}}_{\mu}} - \partial_{\alpha}\frac{\partial\mathcal{L}}{\partial(\partial_{\alpha}\mathcal{B}_{\mu})} \tag{2.4a}$$

$$\mathcal{Q}^{\mu} = \partial\mathcal{L}/\partial\dot{\mathcal{B}}_{\mu} \tag{2.4b}$$

$$\bar{\Pi}^{(\alpha)} = \partial\mathcal{L}/\partial\dot{\psi}_{(\alpha)} \tag{2.4c}$$

$$\Pi^{(\alpha)} = \partial\mathcal{L}/\partial\dot{\bar{\psi}}_{(\alpha)} \tag{2.4d}$$

The momenta $P^{a\mu}$, $Q^{a\mu}$, $\Pi^a_{(\alpha)}$, and $\bar{\Pi}^a_{(\alpha)}$ now are written as follows:

$$P^{a\mu} = F^{a\mu0} + \frac{\kappa}{4\pi}\epsilon^{0\mu\nu}A^a_{\nu} - \frac{c}{\pi}\mathcal{D}_i\mathcal{D}^iF^{a\mu0} - \mathcal{D}_0Q^{a\mu} - \frac{c}{\pi}\mathcal{D}_i\mathcal{D}_0F^{a\mu i} + f^{abc}A^c_0Q^{b\mu} \tag{2.5a}$$

$$Q^{a\mu} = \frac{c}{\pi}\mathcal{D}_0F^{a\mu0} \tag{2.5b}$$

$$\Pi^a_{(\alpha)} = i\left(\frac{a-1}{2}\right)(\gamma_0\psi^a)_{(\alpha)} \tag{2.5c}$$

$$\overline{\Pi}^a_{(\alpha)} = -i\left(\frac{a+1}{2}\right)(\overline{\psi}^a\gamma_0)_{(\alpha)} \tag{2.5d}$$

where the Poisson brackets for the pairs of canonical conjugate variables $(A^a_\mu; P^{a\mu})$, $(B^a_\mu; Q^{a\mu})$, $(\psi; \overline{\Pi})$, and $(\overline{\psi}; \Pi)$ are as usual (Sundermeyer, 1982).

The above equations (2.5) say that the three primary constraints are

$$\Phi^{(0)a}(x) = Q^{a0}(x) \approx 0 \tag{2.6a}$$

$$\Omega^a_{(\alpha)}(x) = \Pi^a_{(\alpha)} - i\left(\frac{a-1}{2}\right)(\gamma_0\psi^a)_{(\alpha)} \approx 0 \tag{2.6b}$$

$$\overline{\Omega}^a_{(\alpha)}(x) = \overline{\Pi}^a_{(\alpha)} + i\left(\frac{a+1}{2}\right)(\overline{\psi}^a\gamma_0)_{(\alpha)} \approx 0 \tag{2.6c}$$

So, the generator of time evolutions of generic functionals, which is a first-class dynamical quantity in the Dirac sense, is given by

$$H_T = \int d^2x\, \mathcal{H}_T \tag{2.7}$$

In equation (2.7), the extended Hamiltonian density $\mathcal{H}_T$ remains defined by

$$\mathcal{H}_T = \mathcal{H}_{\text{can}} + \delta^a\Phi^{(0)}_a + \overline{\lambda}^{(\alpha)}_a\Omega^a_{(\alpha)} + \overline{\Omega}^a_{(\alpha)}\lambda^{(\alpha)}_a \tag{2.8}$$

where δ^a are bosonic Lagrange multipliers and $\lambda^{(\alpha)}_a$ and $\overline{\lambda}^{(\alpha)}_a$ are fermionic.

In equation (2.8), the explicit form of the Hamiltonian $\mathcal{H}_{\text{can}}$ as a functional of fields and momenta is obtained by computing the following expression:

$$\mathcal{H}_{\text{can}} = B^a_\mu P^{a\mu} + \dot{B}^a_\mu Q^{a\mu} + \dot{\overline{\psi}}\Pi + \overline{\Pi}\dot{\psi} - \mathcal{L} \tag{2.9}$$

Now we must analyze the constraint structure, continuing with the Dirac algorithm. When the consistency conditions $\dot{\Omega} = [\Omega, H_T] \approx 0$ on the two fermionic primary constraints is implemented, the Lagrange multipliers $\lambda^a_{(\alpha)}$ and $\overline{\lambda}^a_{(\alpha)}$ are determined:

$$\lambda^a_{(\alpha)} = \gamma_0\gamma^i\mathcal{D}_i\psi^a + im\gamma_0\psi^a + f^{abc}A^b_0\psi^c \tag{2.10a}$$

$$\overline{\lambda}^a_{(\alpha)} = \mathcal{D}_i\overline{\psi}^a\gamma^i\gamma_0 - im\overline{\psi}^a\gamma_0 + f^{abc}A^b_0\overline{\psi}^c \tag{2.10b}$$

On the other hand, the consistency condition on $\Phi_a^{(0)}(x)$ given in (2.6a) gives rise to the following secondary constraints:

$$\Phi^{(1)a} = [\Phi^{(0)a}, H_T] = -P^{a0} + \mathcal{D}_i Q^{ai} \approx 0 \tag{2.11a}$$

$$\begin{aligned}\Phi^{(2)a} &= [\Phi^{(1)a}, H_T] \\ &= -\mathcal{D}_i P^{ai} - \frac{\kappa}{4\pi}\partial_i A_j^a \epsilon^{ij} - f^{abc} B_i^b Q^{ci} \\ &\quad - f^{abc} A_0^b \mathcal{D}_i Q^{ci} + i f^{abc} \bar{\psi}^b \gamma_0 \psi^c \approx 0\end{aligned} \tag{2.11b}$$

and after some algebra it is possible to find

$$\Phi^{(3)a} = [\Phi^{(2)a}, H_T] = -f^{abc} A_0^b \Phi^{(2)c} \tag{2.11c}$$

The result (2.11c) should be expected, because equation (2.11b) is the zero component of the equations of motion.

So, from equation (2.11c) we can see that $\Phi^{(3)a}$ is naturally a weakly zero quantity.

At this point, we must classify the constraints. By computing the brackets among constraints, we can conclude that there are two first-class constraints ($\Phi^{(0)a}$ and $\Phi^{(1)a}$) and the remaining three ($\Phi^{(2)a}$, Ω^a, and $\bar{\Omega}^a$) are of second class. Consequently, as the second-class constraints are odd, we need to find at least a suitable linear combination from these. It is not hard to show that the linear combination among the constraints $\Phi^{(2)a}$, Ω^a, and $\bar{\Omega}^a$ which gives rise to a new first-class constraint is

$$\begin{aligned}\Theta^a(x) &= f^{abc}(\bar{\psi}^b \Pi^c + \bar{\Pi}^b \psi^c) + \mathcal{D}_i P^{ai} + \frac{\kappa}{4\pi}\epsilon^{ij}\partial_i A_j^a \\ &\quad + f^{abc} B_i^b Q^{ci} + f^{abc} A_0^b \mathcal{D}_i Q^{ci} \approx 0\end{aligned} \tag{2.12}$$

Consequently, equations (2.6a), (2.11a), and (2.12) give the three first-class constraints associated with the gauge symmetries of the coupled system. As pointed out above, the only remaining second-class constraints in the model are the fermionic ones, which in the Dirac picture will be treated as strongly equal to zero equations.

At this stage, we are ready to construct the Dirac brackets and carry out the canonical Dirac quantization formalism. As is well known, the Dirac brackets for variables $O_1(x)$ and $O_2(y)$ are defined by

$$\begin{aligned}&[O_1(x), O_2(y)]_{\mathrm{D}} = \\ &[O_1(x), O_2(y)]_{\mathrm{PB}} - [O_1(x), \Omega_a]_{\mathrm{PB}} \Delta^{ab} [\Omega_b, O_2(y)]_{\mathrm{PB}}\end{aligned} \tag{2.13}$$

where the matrix Δ^{ab} is the inverse of the matrix constructed with the elements $[\Omega_a, \Omega_b]_{\rm PB}$ involving only the second-class constraints Ω_a, i.e., $\Delta^{ab}[\Omega_b, \Omega_c]_{\rm PB} = \delta^a_c$, and one finds

$$\Delta = i\begin{pmatrix} 0 & \gamma_0 \\ \gamma_0^T & 0 \end{pmatrix}\delta^{ab}\delta(x - y) \tag{2.14}$$

Using the definition (2.13), we can obtain the Dirac brackets among dynamical variables. We write below only the nonvanishing Dirac brackets which are different from the Poisson brackets. In the present case, they correspond to brackets involving only fermionic dynamical variables. Therefore, the field–field brackets are written

$$[\bar{\psi}^a_{(\alpha)}(x), \psi^b_{(\beta)}(y)]_{\rm D} = -i(\gamma_0)_{(\beta)(\alpha)}\delta^{ab}\delta(x - y) \tag{2.15a}$$

$$[\psi^a_{(\alpha)}(x), \bar{\psi}^b_{(\beta)}(y)]_{\rm D} = -i(\gamma_0)_{(\alpha)(\beta)}\delta^{ab}\delta(x - y) \tag{2.15b}$$

The field–momentum brackets are

$$[\bar{\psi}^a_{(\alpha)}(x), \Pi^b_{(\beta)}(y)]_{\rm D} = \left(\frac{a - 1}{2}\right)\delta_{(\alpha)(\beta)}\delta^{ab}\delta(x - y) \tag{2.16a}$$

$$[\psi^a_{(\alpha)}(x), \bar{\Pi}^b_{(\beta)}(y)]_{\rm D} = -\left(\frac{a + 1}{2}\right)\delta_{(\alpha)(\beta)}\delta^{ab}\delta(x - y) \tag{2.16b}$$

and finally the momentum–momentum brackets are written

$$[\bar{\Pi}^a_{(\alpha)}(x), \Pi^b_{(\beta)}(y)]_{\rm D} = -\frac{i}{4}(a^2 - 1)(\gamma_0)_{(\beta)(\alpha)}\delta^{ab}\delta(x - y) \tag{2.17a}$$

$$[\Pi^a_{(\alpha)}(x), \bar{\Pi}^b_{(\beta)}(y)]_{\rm D} = -\frac{i}{4}(a^2 - 1)(\gamma_0)_{(\alpha)(\beta)}\delta^{ab}\delta(x - y) \tag{2.17b}$$

Looking at equation (2.13), we see that the Dirac brackets and the Poisson brackets for the bosonic variables are identical.

As noted above, the system can be canonically quantified by using the Dirac brackets and taking the second-class constraints as strongly equal to zero equations.

Hence the constrained Hamiltonian system for this higher-derivative theory is described by the total Hamiltonian

$$H_T^* = \int d^2x\, (\mathcal{H}_{\rm can} + \beta^i\Sigma_i) \tag{2.18}$$

where we have renamed with $\Sigma_i(x)$ ($i = 1, 2, 3$) the three first-class constraints given in (2.6a), (2.11a), and (2.12), corresponding to the gauge invariances of the theory under local gauge transformations, and β^i are three arbitrary parameters.

To complete the canonical quantization, the Dirac brackets between pairs of canonical conjugate boson variables (i.e., $[A^a_\mu, P^{b\nu}]$ and $[B^a_\mu, Q^{b\nu}]$) and the brackets defined in (2.15)–(2.17) must be replaced in the equal-time commutators (or anticommutators) according to the rule

$$[O_1(x), O_2(y)]_D \to \frac{1}{i\hbar}[\hat{O}_1\hat{O}_2 - (-1)^{|O_1||O_2|}\hat{O}_2\hat{O}_1]$$

where $|O_i| = 0$ (or 1) when O_i is bosonic (or fermionic).

In the next section use these results to study the diagrammatic of the model.

3. PATH-INTEGRAL QUANTIZATION

The system we are treating has first- and second-class constraints and so the path-integral quantization must be accomplished according to the Faddeev–Senjanovic formalism (Faddeev, 1970; Senjanovic, 1976). Greco *et al.* (1994), for the simpler Abelian case, constructed the partition function for a higher-derivative model. That was done by generalizing the expression given by Faddeev and Senjanovic for the partition function. For the non-Abelian model containing higher-derivative terms, we assume that the partition function in the Hamiltonian formalism is given by

$$\begin{aligned} Z = \int & \mathcal{D}\mathcal{A}_\mu\, \mathcal{D}\mathcal{P}^\mu\, \mathcal{D}\mathcal{B}_\nu\, \mathcal{D}\mathcal{Q}^\nu\, \mathcal{D}\bar{\psi}_{(\alpha)} \\ & \times \mathcal{D}\Pi^{(\alpha)}\, \mathcal{D}\psi_{(\beta)}\, \mathcal{D}\bar{\Pi}^{(\beta)}\, \delta(\Sigma_1)\, \delta(\Sigma_2)\, \delta(\Sigma_3) \\ & \times \delta(f_1)\, \delta(f_2)\, \delta(f_3)\, \det[\Sigma_1, \Sigma_2, \Sigma_3, f_1, f_2, f_3]_D \delta(\Omega_{(\alpha)})\, \delta(\Omega_{(\beta)}) \\ & \times \det[\Omega_{(\alpha)}, \Omega_{(\beta)}] \exp i\left[\int d^3x\, (\dot{B}^a_\mu P^{a\mu} + \dot{B}^a_\nu Q^{a\nu} + \dot{\bar{\psi}}\Pi + \bar{\Pi}\dot{\psi}) - H_T\right] \end{aligned} \tag{3.1}$$

where the functional integration is performed over all the phase-space volume corresponding to the independent dynamical variables $\mathcal{A}_\mu$, $\mathcal{B}_\mu$, $\mathcal{P}_\mu$, and $\mathcal{Q}_\mu$.

In equation (3.1) H_T is the extended Hamiltonian defined in (2.7). The quantities $f_i \approx 0$ are the gauge-fixing conditions, one for each first-class constraint $\Sigma_i(x)$. They are all independent and really restrict the phase space variables to the physical ones, and so the true Hilbert space is obtained. The gauge-fixing conditions, for all the first-class constraints, must satisfy $\det[\Sigma_i, f_j]_D \neq 0$. Moreover, they must be compatible with the equations of motion and satisfy the condition $[f_i, f_j] \approx 0$.

From Greco *et al.* (1994), we note that in the Abelian case the matrix $[\Sigma_i, f_j]_D$ does not depend on the field variables. The non-Abelian model is different and more complicated, even if it does not contain higher derivatives, because the determinant of this matrix, whose matrix elements are constructed from the Dirac brackets among the pairs (Σ_i, f_j), depends on the field variables (Faddeev, 1970).

When higher-derivative terms are present, only under certain assumptions, depending on the gauge-fixing conditions, can this matrix be written as a suitable reversible nonlocal operator. In such a case the determinant of this matrix results in a nonlocal functional linearly dependent of the gauge field $\mathcal{A}_\mu$.

From the path-integral formalism for non-Abelian gauge theories (Faddeev, 1970) it is well known that in the framework of perturbation theory, the determinant of a nonlocal operator M can be written in the integral representation, by using anticommuting scalar functions $\bar{\eta}$ and η. Thus, a new term is added to the partition function (3.1) giving rise to an effective action.

So, in the higher-derivative model under consideration, to obtain a suitable nonlocal operator M, we must choose three particular gauge-fixing conditions. Of course, this requirement does not avoid the arbitrariness in choosing the functions $f_i \approx 0$. For instance, a convenient set of such conditions compatible with the equations of motion and satisfying $\det[\Sigma_i, f_j]_D \neq 0$ for all first-class constraints Σ_i is

$$f_1^a = B_0^a \approx 0 \tag{3.2a}$$

$$f_2^a = \partial_i B^{ai} \approx 0 \tag{3.2b}$$

$$f_3^a = \partial_i A^{ai} \approx 0 \tag{3.2c}$$

As we will see, the gauge-fixing conditions (3.2) allow us to go over to a general covariant gauge in which the nonlocal operator M appearing in the path-integral quantization of Yang–Mills fields takes its well known covariant expression (see, for instance, Faddeev and Slavnov, 1980).

On the other hand, the matrix $[\Sigma_1, \Sigma_2, \Sigma_3, f_1, f_2, f_3]_D$ can be written as follows:

$$[\Sigma_1, \Sigma_2, \Sigma_3, f_1, f_2, f_3]_D = \begin{pmatrix} A & 0 & 0 \\ 0 & M^{ab} & B \\ 0 & 0 & M^{ab} \end{pmatrix} \tag{3.3}$$

where A and M^{ab} are respectively given by

$$A = [\Sigma_1^a, f_1^b] = -\delta^{ab}\delta(x - y) \tag{3.4a}$$

$$M^{ab} = [\Sigma_3^a, f_3^b] = (\delta^{ab}\nabla^2 - f^{abc}A_i^c\partial^i)\delta(x - y) \tag{3.4b}$$

and $B = [\Sigma_3^a, f_2^b]$ is a cumbersome functional depending on the fields. The $\det[\Sigma_1, \Sigma_2, \Sigma_3, f_1, f_2, f_3]_D$ is independent of the functional B.

Therefore, we can write

$$\det[\Sigma_1, \Sigma_2, \Sigma_3, f_1, f_2, f_3]_D = \det A \cdot \det M^{ab} \cdot \det M^{ab}$$

and so the unique dependence on the field is present in the operator M^{ab}, which is reversible in the framework of perturbation theory. As the quantity $\det A$ is independent of the dynamical variables, it is included in the normalization factor appearing in the partition function (3.1). The same thing can be said for the factor $\det[\Omega_{(\alpha)}, \Omega_{(\beta)}]$ written in equation (3.1).

Before constructing the Feynman rules and the diagrammatic, we come again to the original gauge field $\mathcal{A}_\mu$. To do this, we must add to the action in equation (3.1) a term of the form $\int d^3x\, \Lambda^\mu(\mathcal{B}_\mu - \dot{\mathcal{A}}_\mu)$ with arbitrary multipliers Λ^μ and perform the integration on all their possible values.

Consequently, the partition function becomes

$$\begin{aligned} Z = \int & \mathcal{D}\mathcal{A}_\mu\, \mathcal{D}\mathcal{P}^\mu\, \mathcal{D}\mathcal{B}_\nu\, \mathcal{D}\mathcal{Q}^\nu \\ & \times \mathcal{D}\bar{\psi}_{(\alpha)}\, \mathcal{D}\Pi^{(\alpha)}\, \mathcal{D}\psi_{(\beta)}\, \mathcal{D}\bar{\Pi}^{(\beta)}\, \delta(\Sigma_1)\, \delta(\Sigma_2)\, \delta(\Sigma_3) \\ & \times \delta(f_1)\, \delta(f_2)\, \delta(f_3) \det M^{ab} \delta(\Omega_{(\alpha)}) \delta(\Omega_{(\beta)}) \delta(\mathcal{B}_\mu - \dot{\mathcal{A}}_\mu) \\ & \times \exp i\left[\int d^3x\, (B_\mu^a P^{a\mu} + \dot{B}_\nu^a Q^{a\nu} + \dot{\bar{\psi}}\Pi + \bar{\Pi}\dot{\psi}) - H_T\right] \end{aligned} \tag{3.5}$$

Now, by performing the path integral over the fields $\mathcal{B}_\mu$, $\mathcal{P}^\mu$, $\mathcal{Q}^\mu$, $\bar{\Pi}^{(\alpha)}$, and $\Pi^{(\alpha)}$, we find the partition function

$$Z = \int \mathcal{D}\mathcal{A}_\mu\, \mathcal{D}\bar{\psi}_{(\alpha)}\, \mathcal{D}\psi_{(\beta)}\, \delta(f_2)\delta(f_3)(\det M)^2 \exp i[\mathcal{S}_{\text{eff}}] \tag{3.6}$$

where the effective action $\mathcal{S}_{\text{eff}}$ is given by

$$\begin{aligned} \mathcal{S}_{\text{eff}} = \int d^3x \Bigg(& -\frac{1}{4} F_{ij}^a F^{aij} - \frac{1}{2} F_{0i}^a F^{a0i} + \frac{\kappa}{4\pi} \partial_0 A_i^a A_j^a \epsilon^{ij} \\ & - \frac{\kappa}{4\pi} \partial_i A_0^a A_j^a \epsilon^{ij} + \frac{\kappa}{4\pi} \partial_i A_j^a A_0^a \epsilon^{ij} \\ & + \frac{\kappa}{4\pi} f^{abc} A_0^a A_i^b A_j^c \epsilon^{ij} - \frac{c^2}{4\pi} \partial_0 F_{ij}^a \partial^0 F^{aij} \\ & - \frac{c^2}{2\pi} \partial_i F_{0j}^a \partial^i F^{a0j} - \frac{c^2}{4\pi} \partial_i F_{jk}^a \partial^i F^{ajk} \end{aligned}$$

$$
\begin{aligned}
&- \frac{c^2}{2\pi} f^{abc}\partial_i F^a_{jk} A^{bi} F^{cjk} - \frac{c^2}{2\pi} f^{abc}\partial_0 F^a_{ij} A^{b0} F^{cij} - \frac{c^2}{\pi} f^{abc}\partial_i F^a_{0j} A^{bi} F^{c0j} \\
&- \frac{c^2}{4\pi} f^{abc} f^{ade} A^b_0 A^d_0 F^c_{ij} F^{eij} - \frac{c^2}{2\pi} f^{abc} f^{ade} A^b_i A^{di} F^c_{0j} F^{e0j} \\
&- \frac{c^2}{4\pi} f^{abc} f^{ade} A^b_i A^{di} F^c_{jk} F^{ejk} \\
&+ i\,\frac{a+1}{2}\,\bar{\psi}^a \gamma^\mu \partial_\mu \psi_a + i\,\frac{a-1}{2}\,(\partial_\mu \bar{\psi}^a)\gamma^\mu \psi_a \\
&- m\bar{\psi}^a \psi_a - i f^{abc} \bar{\psi}^b \gamma_\mu \psi^c A^{a\mu} \\
&- \frac{c^2}{2\pi}\,[\partial_0\partial_0 A^a_i - 2 f^{abc}\partial_0 A^b_i A^c_0 - f^{abc} A^b_0 \mathcal{D}_i A^c_0]^2 \Bigg)
\end{aligned} \tag{3.7}
$$

In equation (3.7), the last bracket is obtained as a consequence of the Gaussian path integration on the dynamical variable Q_i.

From equation (3.4b) we can see that the nonlocal operator M^{ab} is written in a noncovariant way. Using the Faddeev–Popov trick to go over to a covariant gauge, we can obtain the covariant form defined by the formula $M_L\alpha(x) = \Box\alpha - \partial_\mu[\mathcal{A}_\mu, \alpha]$. To that purpose, we must transfer the integration measure defined on the surface $f_3 = \partial_i\mathcal{A}^i = 0$ to the surface $f_3' = \partial_\mu\mathcal{A}^\mu = 0$, which defines the Lorentz gauge. The same argument can be used on the surface f_2. Moreover, once the path integral over the field $\mathcal{B}_\mu$ is performed in equation (3.5), the surface f_2 becomes the time derivative of the surface f_3.

Subsequently, as usual in non-Abelian gauge theories, also in the higher-derivative case it is convenient to work in a generalized gauge defined by

$$\partial_\mu \mathcal{A}^\mu(x) = \alpha_1(x) \tag{3.8a}$$

$$\partial_\mu \dot{\mathcal{A}}^\mu(x) = \dot{\alpha}_1(x) = \alpha_2(x) \tag{3.8b}$$

with $\alpha_1(x)$ and $\alpha_2(x)$ arbitrary matrices.

Considering that the partition function (3.6) does not depend on α_1 or α_2, as usual we can integrate over both quantities with a Gaussian weight

$$\exp\left[i\lambda_1 \operatorname{tr} \int d^3x\, \alpha_1^2(x) + ic^2\lambda_2 \operatorname{tr} \int d^3x\, \alpha_2^2(x) \right]$$

Moreover, we must write the functional det $M^{ab}(\mathcal{A})$ by using the integral representation

$$\det M = \int \exp\left[i \int d^3x\, \bar{\eta}^a(x) M^{ab}(\mathcal{A}) \eta^b(x) \right] \mathcal{D}\bar{\eta}\, \mathcal{D}\eta \tag{3.9}$$

where $\overline{\eta}(x)$ and $\eta(x)$ are the auxiliary anticommuting scalar functions called the Faddeev–Popov ghost.

Finally, the partition function (3.6) is given by

$$Z = \int \mathcal{D}\mathcal{A}_\mu \, \mathcal{D}\overline{\psi}_{(\alpha)} \, \mathcal{D}\psi_{(\beta)} \, \mathcal{D}\overline{\eta} \, \mathcal{D}\eta \, \exp i[\mathcal{S}^*] \tag{3.10}$$

where the extended action $\mathcal{S}^*$ is written

$$\mathcal{S}^* = \mathcal{S}_{\text{eff}} - \int d^3x \left[\frac{\lambda_1}{2} (\partial_\mu A^{a\mu})^2 + c^2 \frac{\lambda_2}{2} (\partial_\mu \dot{A}^{a\mu})^2 + \frac{1}{2} \overline{\eta}_i^a \Box \eta_i^a - \frac{1}{2} f^{abc} \overline{\eta}_i^a A^{b\mu} \partial_\mu \eta_i^c \right] \tag{3.11}$$

From equation (3.10) we can see that the quantum problem remains defined in terms of a path integral, in which the independent fields are the original gauge field $\mathcal{A}_\mu$, the matter Dirac spinor field ψ, and the unphysical ghost fields $\overline{\eta}$ and η. Consequently, it is possible now to apply diagrammatic techniques defining proper Feynman rules for propagators and vertices corresponding to these fields.

4. DIAGRAMMATIC AND FEYNMAN RULES

Looking at equation (3.11) and taking into account expression (3.7) for $\mathcal{S}_{\text{eff}}$, we can easily recognize the propagators defined by the quadratic part of the Lagrangian density and the remaining part of it can be represented by vertices.

The propagator of the fermionic field ψ is the usual one. Therefore, it is more interesting to analyze the gauge field propagator in which the higher-derivative feature of the model is exposed.

The action $\mathcal{S}^*$ can be written in pieces as follows:

$$\mathcal{S}^* = \mathcal{S}_{\text{p}}^*(\mathcal{A}_\mu) + \mathcal{S}_{\text{v}}^*(\mathcal{A}_\mu) + \mathcal{S}^*(\psi) + \mathcal{S}_{\text{int}}^*(\mathcal{A}_\mu, \psi) + \mathcal{S}_{\text{ghost}}^* \tag{4.1}$$

where $\mathcal{S}_{\text{p}}^*(\mathcal{A}_\mu)$ defines the gauge field propagator; $\mathcal{S}_{\text{v}}^*(\mathcal{A}_\mu)$ defines the different vertices whose legs are gauge fields; $\mathcal{S}^*(\psi)$ defines the fermionic field propagator; $\mathcal{S}_{\text{int}}^*(\mathcal{A}_\mu, \psi)$ defines the usual vertex $\overline{\psi}\psi\mathcal{A}_\mu$; and finally $\mathcal{S}_{\text{ghost}}$ contains the ghost field propagator and the well-known vertex $\overline{\eta}\mathcal{A}\partial\eta$, which is linear in momentum.

For instance, the first term on the right-hand side of equation (4.1) corresponding to the gauge field propagator can be written

$$\mathcal{S}_p^* = \int d^3x[A^a_\mu(D^{-1})^{\mu\nu}A^a_\nu] \tag{4.2}$$

The 3×3 matrix $(D^{-1})^{\mu\nu}$ defined in equation (4.2) is the inverse of the propagator of the gauge field $\mathcal{A}_\mu$. It is Hermitian and nondegenerate and it can be invertible. So, the propagator $D_{\mu\nu}(k)$, in the momentum space, can be straightforwardly evaluated. The general case for nonzero topological mass of the gauge field is very tedious to compute and does not bring anything new. It is possible, however, to obtain the expression for the propagator in the $\kappa = 0$ case, which gives useful information. The $\det[D^{-1}(k)]$ is given by

$$\det[D^{-1}(k)] = \epsilon(1 - c^2\epsilon) \times \Delta(k_0, \mathbf{k}) \tag{4.3}$$

where $\Delta(k_0, \mathbf{k})$ is the function

$$\Delta = \lambda\epsilon^2(1 - c^2\mathbf{k}^2) + c^2(\mathbf{k}^2 - \epsilon)^2[\lambda(\mathbf{k}^2 - \epsilon) - \mathbf{k}^2(1 - c^2\mathbf{k}^2)] \tag{4.4}$$

and $k_1^2 + k_2^2 - k_0^2 = \mathbf{k}^2 - k_0^2 = \epsilon$ and $\lambda_1 + c^2\lambda_2k_0^2 = \lambda$.

Now, by computing the matrix elements $D_{\mu\nu}(k)$, we find the gauge field propagators

$$D_{00} = \frac{(1 - c^2\epsilon)(\epsilon - \mathbf{k}^2) + \lambda\mathbf{k}^2}{\Delta} \tag{4.5a}$$

$$D_{0i} = D_{i0} = \frac{(\lambda - 1 + c^2\mathbf{k}^2)k_0k_i}{\Delta} \tag{4.5b}$$

$$D_{ij} = \frac{1}{\epsilon(1 - c^2\epsilon)}\left(g_{ij} + \frac{k_ik_j\Lambda}{\Delta}\right) \tag{4.5c}$$

where $\Lambda(k_0, \mathbf{k})$ is given by

$$\Lambda = \epsilon(1 - c^2\mathbf{k}^2)(\lambda - 1 + c^2\mathbf{k}^2) + c^2(\mathbf{k}^2 - \epsilon)[\lambda(\mathbf{k}^2 - \epsilon) - \mathbf{k}^2(1 - c^2\mathbf{k}^2)] \tag{4.6}$$

Looking at equations (4.4) and (4.6), we see that in the limit $c^2 \to 0$, the above functions take the values $\Delta = \lambda_1\epsilon^2$ and $\Lambda = \epsilon(\lambda_1 - 1)$. Therefore, in that limit, the covariant expression of the gauge field propagator is given by $D_{\mu\nu} = g_{\mu\nu}/\epsilon + (1 - \lambda_1^{-1})k_\mu k_\nu/\epsilon^2$. This form, depending on the unique parameter λ_1, is usual in a non-Abelian gauge theory written in a general covariant gauge.

At this stage we must look at the convenience of adding higher-derivative terms in the Lagrangian density. The first remark is that for large momentum, the propagator behaves like $\sim 1/k^4$.

This guarantees that in those diagrams in which the propagator of the gauge field occurs, the ultraviolet behavior is improved. In the perturbative framework, we are going to consider one-loop diagrams, which in the model without higher-derivative terms have a superficial degree of divergence. Such diagrams are, for instance, the correction to the fermion line $\Sigma(p)$ and the vertex correction $V_\rho(p, q)$, whose analytical expressions have respectively the form

$$\Sigma(p) \sim \int \frac{d^3k}{(2\pi)^3} \frac{\gamma_\mu(\gamma\cdot p - \gamma\cdot k - m)\gamma_\nu}{(p - k)^2 + m^2} \times D_{\mu\nu}(k) \tag{4.7}$$

$$V_\rho(p, q) \sim \int \frac{d^3k}{(2\pi)^3} \gamma_\mu \frac{(\gamma\cdot p + \gamma\cdot k - m)}{(p + k)^2 + m^2} \gamma_\rho \times \frac{(\gamma\cdot q + \gamma\cdot k - m)}{(q + k)^2 + m^2} \gamma_\nu \times D_{\mu\nu}(k) \tag{4.8}$$

If we analyze these diagrams, we can see that for large momentum of the gauge field, the integrals in equations (4.7) and (4.8) behave as $\sim\int dk/k^3$ and $\sim dk/k^4$, respectively. Consequently, the new gauge field propagator has an ultraviolet behavior so that the Feynman integrals (4.7) and (4.8) give convergent results.

Finally, we briefly comment upon the part $\mathcal{S}_V^*$ of the action written in (4.1), which defines the different vertices of the model with gauge field legs. Due to the introduction of higher-derivative terms, the appearance of new vertices in the model is the price we must pay. Looking at the expression (3.7) for $\mathcal{S}_{\text{eff}}$, in addition to the usual three-leg vertex in the gauge field AAA, we can see that there are new vertices containing more legs in the gauge field and some of them with momentum insertion. All the different vertices defined in $\mathcal{S}_V^*$ are present in terms of the following type: $c^2 f^{abc} f^{ade} A^b F^c A^d F^e$, $c^2 f^{abc} \partial F^a A^b F^c$, and $c^2 \partial F^a \partial F^a$.

The above results and comments do not solve the problem of regularization and renormalization of the model. Simply, it was found that the new propagator (4.5) has a better ultraviolet behavior because for large momentum it behaves like k^{-4} and so we can gain two powers with respect to the usual propagator. Therefore, we can conclude that the inclusion of higher-derivative terms in the Lagrangian improves the behavior of propagators at large momentum, rendering the model less divergent.

On the other hand, it is known that the dimensional regularization method is problematic in a field theory containing the volume form $\epsilon^{\mu\nu\rho}$. In these cases, other gauge-invariant regularization methods, for instance, the Pauli–Villars procedure, must be used (Alvarez-Gaume *et al.*, 1990). Moreover, it is believed that the Chern–Simons field theories belong to the category of finite theories, that is, theories containing a finite number of divergent diagrams. Consequently, at this stage, in order to solve completely the problem, it only remains to regularize a renormalizable model. We do not complete the procedure here because it is carried out following conventional methods.

Before finishing the analysis, the unitarity problem deserves comment. It is well known (Hawking, 1987) that in quantum field theories described by Lagrangians containing higher-derivative terms unitarity can be violated. This occurs when ghost states with negative norm are present. In a previous paper (Foussats *et al.*, 1995), where an Abelian higher-derivative model was analyzed, the unitarity problem was treated carefully at least at tree level. In the non-Abelian case the same discussion holds. Therefore, following the steps given in Foussats *et al.* (1995; see also 't Hooft and Velman, 1973), we must consider first the gauge field propagator $D_{\mu\nu}(k)$. Next, we consider the 3×3 matrix residue $K^{R}_{\mu\nu}(k)$ obtained from the matrix $D_{\mu\nu}$ by leaving out the poles. The matrix residue $K^{R}_{\mu\nu}(k)$ is Hermitian and can be diagonalized and has three different nonzero eigenvalues. Consequently (Faddeev and Slavnov, 1980), a set (α) of real currents $J^{(\alpha)}_{\mu}(k)$ can be defined, one for every nonzero eigenvalue. When all the eigenvalues of the matrix residue at the pole are positive the normalization is given by

$$J^{(\alpha)}_{\mu}(k)K^{R}_{\mu\nu}(k)J^{(\alpha)}_{\nu}(k) = +1 \tag{4.9}$$

When the matrix residue has a negative eigenvalue at the pole, it corresponds to states with negative norm (i.e., the unitarity is lost) and they are physically unacceptable. So, by assuming the existence of a positive metric Hilbert subspace stable under the time evolution, in order to recover the unitarity, the normalization in equation (4.9) must be done with a minus one. This trick to retrieve the unitarity of a theory is usually known as the indefinite metric prescription.

5. CONCLUSIONS

The conclusions are very simple. Starting from a singular second-order Lagrangian, we found the classical generalized Hamiltonian formalism for a non-Abelian CS gauge theory described by means of a higher-order Lagrangian coupled to fermionic matter. Next, the canonical quantization was carried out by following as closely as possible the canonical Dirac algorithm valid for usual constrained Hamiltonian systems. Once the set of constraints was classified into first class and second class and the Hamiltonian of the system as first-class dynamical quantity was found, we were ready to construct the path-integral formalism.

Then the idea was to apply the path-integral method in the framework of perturbation theory in order to obtain information about the behavior of some Feynman integrals. The aim was to know if the convergence of the model is improved or not when higher-derivative terms are added to the

Lagrangian. We showed that the new propagator of the non-Abelian gauge field has an ultraviolet behavior such that the higher-derivative model is less divergent than the usual one.

The path-integral quantization method is also very interesting because it can satisfactorily solve the partition function in the Hamiltonian formalism by using a natural generalization of the Faddeev–Senjanovic method. This is possible because the equation (3.5) for the partition function is written in the extended phase space in terms of H_T. The Hamilton equations allows us to come back to the original action as a functional depending only on the gauge field $\mathcal{A}_\mu$.

Moreover, we can find a suitable set of compatible gauge-fixing conditions which satisfy $\det[\Sigma_i, f_j]_D \neq 0$. This determinant has just the gauge field dependence which permits us to use the Faddeev–Popov trick to go over to a general covariant gauge and to introduce the ghost auxiliary field variables.

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REFERENCES

Alvarez-Gaume, L., Labastida, J. M. F., and Ramallo, A. V. (1990). *Nuclear Physics B*, **334**, 103.

Anderson, P. W. (1987). *Science*, **235**, 1196.

Arovas, D. P., Schrieffer, J. R., Wilczek, F., and Zee, A. (1985). *Nuclear Physics B*, **251**, 117.

Avdeev, L., Grigoryev, G., and Kazakov, D. (1992). *Nuclear Physics B*, **382**, 561.

Babinovici, E., Schwimmer, A., and Yankielowicz, S. (1984). *Nuclear Physics B*, **248**, 523.

Bednorz, G., and Müller, K. A. (1986). *Zeitschrift für Physik B*, **64**, 188.

Bowick, M. J., Karabali, D., and Wijewardhana, L. C. R. (1986). *Nuclear Physics B*, **271**, 417.

Cortés, J. L., Gamboa, J., and Velázquez, L. (1992). *Physics Letters B*, **286**, 105.

Cortés, J. L., Gamboa, J., and Velázquez, L. (1994). *International Journal of Modern Physics A*, **9**, 953.

Deser, S., Jackiw, R., and Templeton, S. (1982a). *Physical Review Letters*, **48**, 975.

Deser, S., Jackiw, R., and Templeton, S. (1982b). *Annals of Physics*, **140**, 372.

Deser, S., Jackiw, R., and Templeton, S. (1988). *Annals of Physics*, **195**, 406.

Dirac, P. A. M. (1964). *Lectures on Quantum Mechanics*, Yeshiva University Press, New York.

Dzyaloshinskii, I., Polyakov, A. M., and Wiegmann, P. B. (1988). *Physics Letters A*, **127**, 112.

Ellis, R. (1975). *Journal of Physics A: Mathematical and General*, **8**, 496.

Faddeev, L. D. (1970). *Theoretical and Mathematical Physics*, **1**, 1.

Faddeev, L. D., and Slavnov, A. A. (1980). *Gauge Fields, Introduction to Quantum Theory*, Benjamin/Cummings, Redwood City, California.

Foussats, A., Repetto, C., Zandron, O. P., and Zandron, O. S. (1992). *Classical and Quantum Gravity*, **9**, 2217.

Foussats, A., Manavella, E., Repetto, C., Zandron, O. P., and Zandron, O. S. (1995). *International Journal of Theoretical Physics*, **34**, 13.

Goldhaber, A. S. (1982). *Physical Review Letters,* **49**, 905.
Greco, A., Repetto, C., Zandron, O. P., and Zandron, O. S. (1994). *Journal of Physics A: Mathematical and General,* **27**, 239.
Hagen, C. R. (1984). *Annals of Physics,* **157**, 342.
Hagen, C. R. (1985). *Physical Review D,* **31**, 2135.
Halperin, B. I. (1984). *Physical Review Letters,* **52**, 1583, Erratum, **52**, 2390.
Hawking, S. W. (1987). In *Quantum Field Theory and Quantum Statistics,* Vol. 2, I. A. Batalin, C. J. Isham, and G. A. Volkovisky, eds., Adam Hilger, Bristol.
Jackiw, R. (1987). *Quantum Field Theory and Quantum Statistics,* Vol. 2, I. A. Batalin *et al.*, eds., Adam Hilger, Bristol.
Jackiw, R., and Redlich, A. N. (1983). *Physical Review Letters,* **50**, 555.
Jackiw, R., Bak, D., and Pi, So-Young (1994). *Physical Review D,* **49**, 6778.
Kerstyen, P. H. M. (1988). *Physics Letters A,* **134**, 25.
Lee, B. W., and Zinn-Justin, J. (1972). *Physical Review D,* **5**, 3121.
Leibbrandt, G., and Martin, C. P. (1992). *Nuclear Physics B,* **377**, 593.
Leibbrandt, G., and Martin, C. P. (1994). *Nuclear Physics B,* **416**, 351.
Leon, M. D., and Rodriguez, P. R. (1985). *Generalized Classical Mechanics and Field Theory,* North-Holland, Amsterdam.
Li, Zi-ping (1991). *Journal of Physics A: Mathematical and General,* **24**, 4261.
Lin, Qiong-gui, and Ni, Guang-jiong (1990). *Classical and Quantum Gravity,* **7**, 1261.
Lüscher, M. (1989). *Nuclear Physics B,* **326**, 557.
Matsuyama, T. (1989). *Physics Letters B,* **228**, 99.
Matsuyama, T. (1990a). *Journal of Physics A: Mathematical and General,* **23**, 5241.
Matsuyama, T. (1990b). *Progress of Theoretical Physics,* **84**, 1220.
Nesterenko, V. V. (1989). *Journal of Physics A: Mathematical and General,* **22**, 1673.
Odintsov, S. (1992). *Zeitschrift für Physik C,* **54**, 527.
Ostrogradski, M. (1850). *Memoirs de l'Academie St. Petersbourg,* **1**, 385.
Panigrahi, P. K., Roy, S., and Scherer, W. (1988a). *Physical Review D,* **38**, 3199.
Panigrahi, P. K., Roy, S., and Scherer, W. (1988b). *Physical Review Letters,* **61**, 2827.
Polyakov, A. M. (1988). *Modern Physics Letters A,* **3**, 325.
Semenoff, G. W. (1988). *Physical Review Letters,* **61**, 517.
Senjanovic, P. (1976). *Annals of Physics,* **100**, 227.
Shonfeld, J. (1981). *Nuclear Physics B,* **185**, 157.
Siegel, W. (1979). *Nuclear Physics B,* **156**, 135.
Slavnov, A. A. (1971). *Nuclear Physics B,* **31**, 301.
Slavnov, A. A. (1977). *Theoretical and Mathematical Physics,* **33**, 977.
Slavnov, A. A. (1981). Symmetry preserving regularization for gauge and supergauge theories, in *Superspace and Supergravity,* S. W. Hawking and M. Rocek, eds., Cambridge University Press, Cambridge.
Sundermeyer, K. (1982). *Constrained Dynamics,* Springer-Verlag, Berlin.
'T Hooft, G., and Velman, M. (1973). *Diagramar,* CERN, Geneva.
Van Nieuwenhuizen, P. (1985). *Physical Review D,* **32**, 872.
Wiegmann, P. B. (1988). *Physical Review Letters,* **60**, 821.
Wilczek, F. (1982a). *Physical Review Letters,* **48**, 1144.
Wilczek, F. (1982b). *Physical Review Letters,* **49**, 957.
Witten, E. (1988). Preprint IAS IASSNS-HEP 88/3.
Wu, Y. S. (1984a). *Physical Review Letters,* **52**, 2103.
Wu, Y. S. (1984b). *Physical Review Letters,* **53**, 111.